Hydropower Plants: Generating and Pumping Units
Solved Problems: Series 4

1 HYDROPOWER PLANT EQUIPPED WITH KAPLAN TURBINES

The Gezhouba power plant is located in the Hubei province, in China, where the frequency of the electrical grid is equal to f_{grid} = 50 Hz. It is equipped with 2 Kaplan turbines of 176 MW and 5 Kaplan turbines of 129 MW. In this problem, we will investigate the 176 MW units. A cut-view of the Kaplan unit is given in Figure 1.

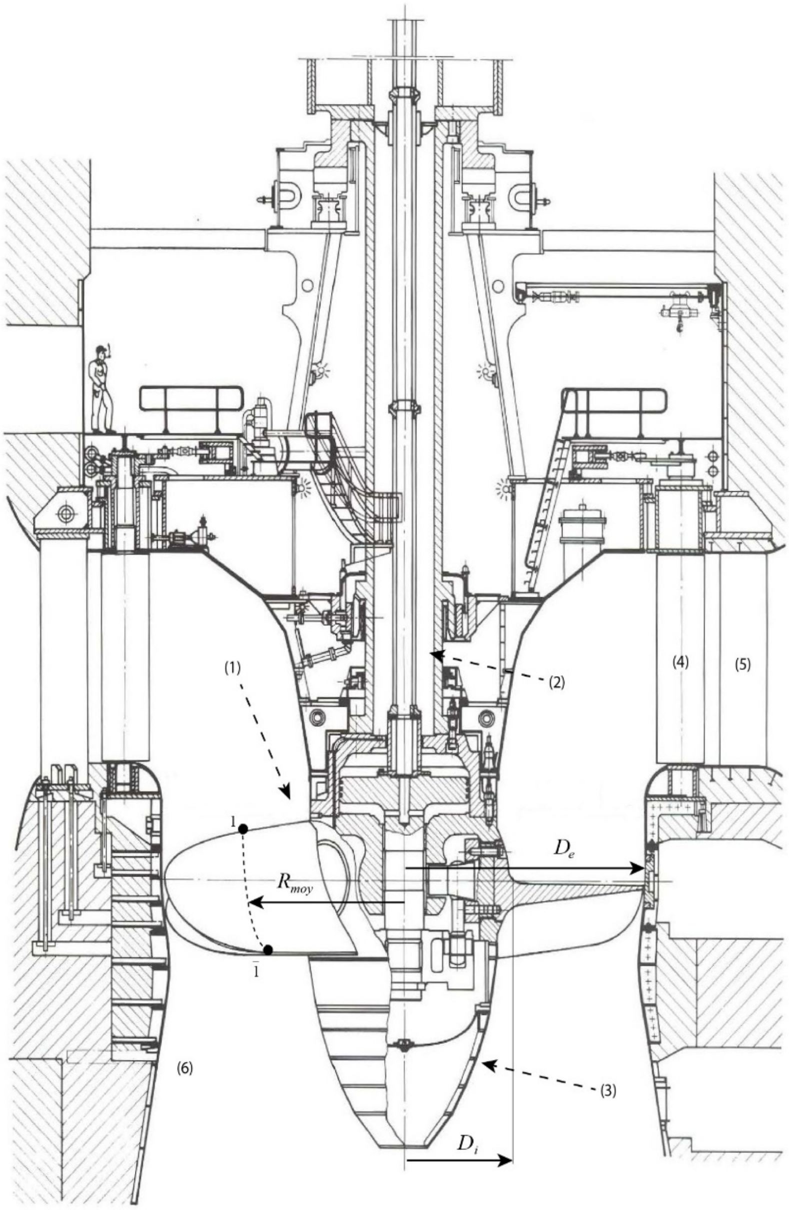

Figure 1: Kaplan turbine unit from Gezhouba power plant.

1. Find the adequate name for the power plant components numbered in Figure 1:

Number	Name
(1)	*Runner*
(2)	*Shaft*
(3)	*Hub*
(4)	*Guide vanes*
(5)	*Stay vanes*
(6)	*Draft tube*

2. Compute the specific potential energy of the installation for an upstream reservoir level of $Z_B = 45$ m and a downstream reservoir level of $Z_{\bar{B}} = 25$ m. The value of the gravitational constant in the Gezhouba power plant is $g = 9.794$ m s^{-2}.

$$gH_B - gH_{\bar{B}} = g(Z_B - Z_{\bar{B}}) = 195.88 \text{ Jkg}^{-1}$$

3. For a nominal discharge of $Q = 1130$ m^{3}s^{-1}, the head losses of the installation have been measured and are equal to $\sum gH_r = 13.48$ J kg^{-1}. Compute the available specific energy of the turbine. Deduce the net head H of the turbine.

$$E = gH_B - gH_{\bar{B}} - \sum gH_r = 182.4 \text{ J kg}^{-1}$$

$$H = \frac{E}{g} = 18.62 \text{ m}$$

This means that the losses in the installation can be represented as an "loss" of head of 1.38 m.

4. For this unit, the generator has a pole number equal to $Z_0 = 110$. Compute the runner frequency n and the specific speed ν of the runner.

$$n = \frac{2 f_{grid}}{Z_0} = 0.91 \text{ Hz}$$

$$\omega = 2\pi n = 5.712 \text{ rad s}^{-1}$$

$$\nu = \frac{\omega\sqrt{Q}}{\sqrt{\pi}\,(2E)^{\frac{3}{4}}} = 1.298$$

5. Compute P_h, the hydraulic power. The value of the water density ρ is 998 kg m^{-3}.

$$P_h = \rho\, Q\, E = 205.7 \text{ MW}$$

6. We assume that the energy efficiency for this turbine is $\eta_e = 92$ %. Compute the transformed (or supplied) specific energy E_t.

$$E_t = \eta_e E = 167.81 \text{ J kg}^{-1}$$

7. Compute the torque experienced by the runner shaft T_t.

We can neglect the leakage flow losses, which means that the volumetric efficiency $\eta_q = 1.0$*, and therefore the transferred discharge is* $Q_t = Q$

$$P_t = \rho\, Q\, E_t = 189.25 \text{ MW}$$

$$P_t = \omega T_t \Rightarrow T_t = \frac{P_t}{\omega} = 33.13\times 10^6 \text{ Nm}$$

8. Compute the mechanical efficiency η_{me} and global machine efficiency. Neglect the generator losses.

Using the output Power given in the description of the exercise, and the transferred power computed in question 7:

$$\eta_{me} = \frac{P}{P_t} = \frac{176}{189.25} = 0.93$$

The overall efficiency corresponds to the output power over the hyraulic power, which represents the maximum available power:

$$\eta = \frac{P}{P_h} = 0.86$$

Another way of computing the overall efficiency is to represent it as the product of all the efficiencies. In this case, there are no generator or leakage losses, therefore:

$$\eta = \eta_{me}\eta_e$$

9. The streamline $1-\overline{1}$ can be approximated as an open cylinder with a mean radius R_m. The internal and external diameters are equal to D_i = 4.520 m and D_e = 11.3 m, respectively. Compute the peripheral runner speed U_1 and $U_{\overline{1}}$.

$$U_1 = \omega R_m = 2\pi n R_m = 2\pi n \frac{D_e + D_i}{4} = 22.61 \text{ ms}^{-1}$$

And, knowing that the runner speed is the same at the inlet and outlet:

$$U_{\overline{1}} = U_1 = 22.61 \text{ ms}^{-1}$$

10. By considering that the flow at the runner outlet is purely axial, compute Cu_1 the peripheral component of the absolute velocity at the runner inlet.

By solving the Euler equation and considering a purely axial flow ($Cu_{\overline{1}} = 0 \text{ ms}^{-1}$):

$$E_t = U_1 Cu_1 - U_{\overline{1}} Cu_{\overline{1}} = U_1 Cu_1$$

Which means that:

$$Cu_1 = \frac{E_t}{U_1} = 7.42 \text{ ms}^{-1}$$

11. Compute the meridional components of the absolute velocity Cm_1 et $Cm_{\overline{1}}$.

We first compute the inlet and outlet areas:

$$A_1 = A_{\overline{1}} = \frac{\pi(D_e^2 - D_i^2)}{4} = 84.24 \,\text{m}^2$$

Then, we use the fact that the flow in a Kaplan turbine is axial at both the inlet and outlet. This means that the discharge can be expressed as a function of the meridional component of the absolute velocity:

$$C_{m_1} = \frac{Q}{A_1} = 13.41 \text{ ms}^{-1}$$

$$C_{m_{\bar{1}}} = \frac{Q}{A_{\bar{1}}} = 13.41 \text{ ms}^{-1}$$

Which is in adequation with the prinicple of mass flow conservation.

12. From the previous results, compute the angles α_1 and β_1 at the runner inlet, and $\alpha_{\bar{1}}$ and $\beta_{\bar{1}}$ at the runner outlet.

Using the trigonometric relations in the inlet and outlet velocity triangles:

$$\alpha_1 = \tan^{-1}\left(\frac{Cm_1}{Cu_1}\right) = 61.04^\circ$$

$$\beta_1 = \tan^{-1}\left(\frac{Cm_1}{U_1 - Cu_1}\right) = 41.44^\circ$$

$$\alpha_{\bar{1}} = 90^\circ$$

$$\beta_{\bar{1}} = \tan^{-1}\left(\frac{Cm_{\bar{1}}}{U_{\bar{1}} - Cu_{\bar{1}}}\right) = 30.67^\circ$$

°

13. Finally, sketch the corresponding velocity triangles at the runner inlet and outlet.

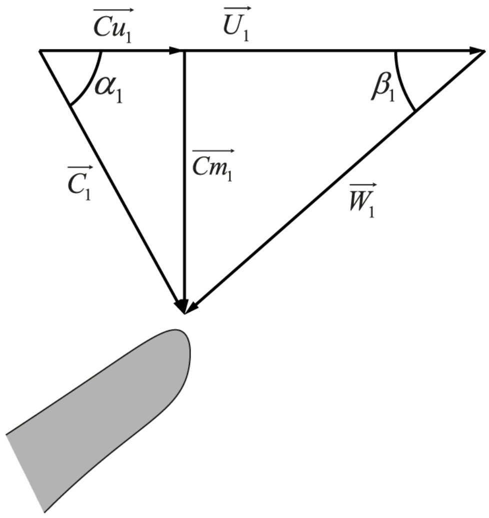

Figure 4: Velocity triangle at runner inlet, with angles drawn on scale

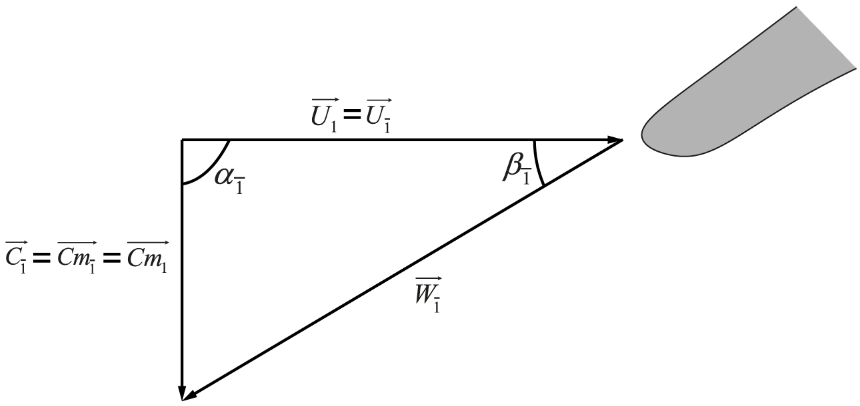

Figure 5: Velocity triangle at runner outlet, with angles drawn on scale

2 FUNDAMENTAL STUDY FOR TRANSFORMED SPECIFIC ENERGY

Here, the fundamentals of hydraulic power plants and the calculation of the transformed specific energy E_t are studied. The general sketch of a hydraulic power plant with a pump-turbine unit is shown in Figure 2, where the numeric values of the operating condition are shown as well. The pump-turbine is operated in turbine mode at the best efficiency point. The points 1 and $\bar{1}$ correspond to the inlet and the outlet of the turbine, respectively. For gravity acceleration and density, use the following values:

$$g = 9.81 \text{ m s}^{-2},\ \rho = 1'000 \text{ kg m}^{-3}$$

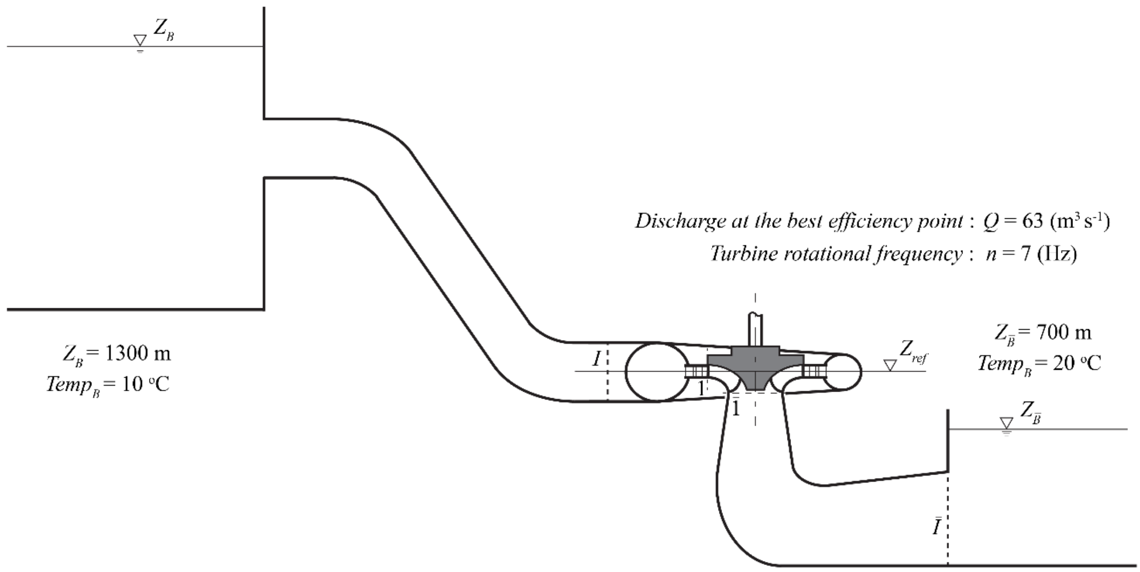

Figure 2: Layout of a pump-turbine installation

1) Assuming that an atmosphere pressure p_a is constant, express the potential specific energy $E_{potential}$ by g, Z_B and $Z_{\bar{B}}$. Then, calculate its value.

$$E_{potential} = g\left(Z_B - Z_{\bar{B}}\right) \cong 5886 \,\text{J kg}^{-1}$$

2) Assuming a constant atmospheric pressure is a good first approximation. However, in reality, the atmosphere pressure changes with altitude and temperature. Considering the difference of atmosphere pressure between both reservoirs, express the potential specific energy $E_{potential}$ as a function of g, ρ, Z_B, $Z_{\bar{B}}$, p_{a_B} and $p_{a\_\bar{B}}$. Then, calculate the value of $E_{potential}$.

   The atmospheric pressure can be calculated as a function of the altitude h (in meters) and the temperature T (in °C) using the following equation:

$$p_a = p_0\left(1 - \frac{0.0065h}{T_0 + 273.15}\right)^{5.257}$$

$$p_0 = 101.3\,\text{kPa},\quad T_0 = T + 0.0065h$$

$$E_{potential} = \left\{\left(gZ_B + \frac{p_{a\_B}}{\rho}\right) - \left(gZ_{\bar{B}} + \frac{p_{a\_\bar{B}}}{\rho}\right)\right\} \cong 5879.37\,\text{J kg}^{-1}$$

3) Express the available specific energy E selecting the necessary variables among $E_{potential}$, $gH_{rB \div I}$, $gH_{rI \div 1}$, $gH_{r\bar{1} \div \bar{I}}$, and $gH_{r\bar{I} \div \bar{B}}$.

$$E = E_{potential} - gHr_{B \div I} - gHr_{\bar{I} \div \bar{B}}$$

4) Express the transformed specific energy E_t selecting the necessary variables among $E_{potential}$, $gH_{rB \div I}$, $gH_{rI \div 1}$, $gH_{r\bar{1} \div \bar{I}}$, and $gH_{r\bar{I} \div \bar{B}}$.

$$E_t = E_{potential} - gHr_{B \div I} - gHr_{\bar{I} \div \bar{B}} - gHr_{I \div 1} - gHr_{\bar{1} \div \bar{I}}$$

5) The transformed power P_t is defined by $P_t = \rho Q_t E_t$, where Q_t is the discharge passing through the turbine, which is lower than the discharge Q. Give an explanation of this difference.

The discharge leaks through the clearance between the rotational and stationary parts (turbine and casing), therefore the discharge passing through the turbine Q_t is lower than the discharge Q.

6) The transformed power P_t is related to the output power P as $P_t = \dfrac{1}{\eta_{me}} P$ (with η_{me} the mechanical efficiency defined by $\eta_{me} = \eta_m \eta_{rm}$, where η_m is the efficiency of the bearing and η_{rm} the efficiency of the disc friction). Express the transformed power P_t by the mechanical efficiency η_{me}, global efficiency η, density ρ, discharge Q, available energy E.

Using $P = \eta \rho Q E$ *:*

$$P_t = \frac{\eta}{\eta_{me}} \rho Q E$$

7) Introducing the volumetric efficiency and the energetic efficiency defined as $\eta_q = \dfrac{Q_t}{Q}$ and $\eta_e = \dfrac{E_t}{E}$ respectively, express the global efficiency η by η_e, η_q, η_m, and η_{rm}.

$$\eta = \eta_m \eta_{rm} \eta_e \eta_q$$

8) Assuming that the losses $gH_{rB \div I} + gH_{r\bar{I} \div \bar{B}}$ correspond to 5% of the potential specific energy, calculate the hydraulic power P_h.

$$P_h = \rho Q E = \rho Q \times 0.95 \times E_{potential} = 352.28\,\text{MW}$$

For the calculation of the transformed specific energy E_t, the velocity triangle representing the relationship of the discharge velocity with the turbine rotational velocity and the Euler equation play a decisive role. The schematics of the pump-turbine and an example of the velocity triangle are shown in Figure 3. In this section, we set the values of k_{Cu1e} and $k_{Cu\bar{1}e}$ such that $k_{Cu1e} = 1$ and $k_{Cu\bar{1}e} = \dfrac{1}{2}$. If necessary, use the following values: $D_{1e} = 4.0\,\text{m}$, $D_{\bar{1}e} = 1.6\,\text{m}$, $B = 0.234\,\text{m}$

$$E_t = k_{Cu1e}\left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right) - k_{Cu\bar{1}e}\left(\vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e}\right) = \left(\vec{C}_{1e} \cdot \vec{U}_{1e}\right) - \frac{1}{2}\left(\vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e}\right)$$

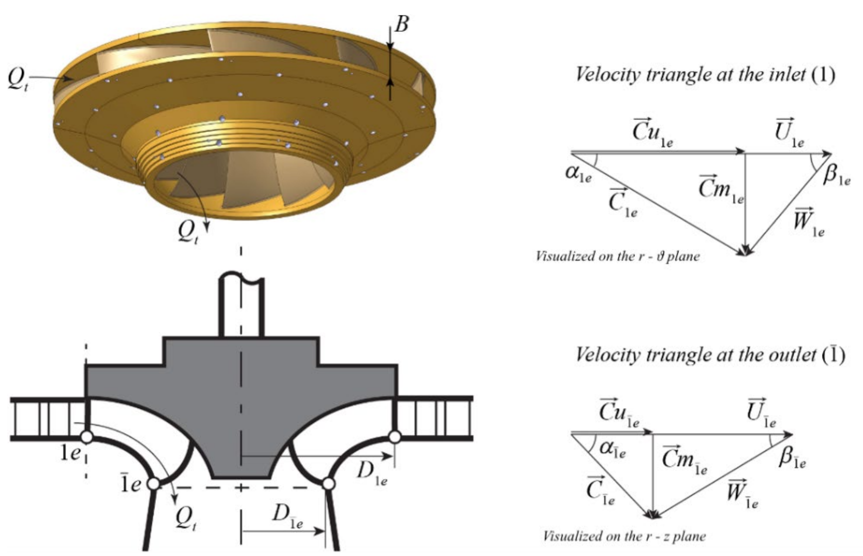

Figure 3: Velocity triangle of the pump-turbine in turbine mode

9) Referring to the vectorial relationship in Figure 3, deduce the scalar form of the Euler equation using the necessary variables among Cu_{1e}, Cm_{1e}, U_{1e}, $Cu_{\bar{1}e}$, $Cm_{\bar{1}e}$, and $U_{\bar{1}e}$.

Using the given values of k_{Cu1e} and $k_{Cu\bar{1}e}$:

$$E_t = \vec{C}_{1e} \cdot \vec{U}_{1e} - \frac{1}{2}\vec{C}_{\bar{1}e} \cdot \vec{U}_{\bar{1}e} = Cu_{1e}U_{1e} - \frac{1}{2}Cu_{\bar{1}e}U_{\bar{1}e}$$

10) Calculate the meridional velocity component Cm_{1e} and the turbine rotational velocity U_{1e} assuming a volumetric efficiency $\eta_q = 0.99$.

Knowing that the inletflow is radial: $A_{1e} = \pi D_{1e} B$

Thus:

$$Cm_{1e} = \frac{Q_t}{A_{1e}} = \frac{\eta_q Q}{\pi D_{1e} B} = \frac{0.99 \times 63}{\pi \times 4.0 \times 0.234} = 21.21\,\mathrm{m\,s^{-1}}$$

$$U_{1e} = \frac{1}{2} D_{1e}\omega \cong 87.96\,\mathrm{m\,s^{-1}}$$